

Dynamic Graph Clustering with Graph Neural Networks: spectral end-to-end differentiable optimization of temporal modularity

Background

PROBLEM

Node clustering (*community detection*) is a crucial task for many applications [1,2] to obtain coarse-grained graphs.

In dynamic graphs with rich, high-dimensional node or edge features, it requires **jointly modeling** structure- and attribute-based signals, while also considering temporal dynamics at micro, macro and mesoscales.

RELATED WORK

A compendium of techniques has been proposed for community detection [2], including spectral, inferential, and greedy heuristic optimization approaches; and more recently, neural networks for graphs, each with their very own (dis)advantages – and often **conflicting definitions** of what constitutes a *community* itself.

LIMITATIONS

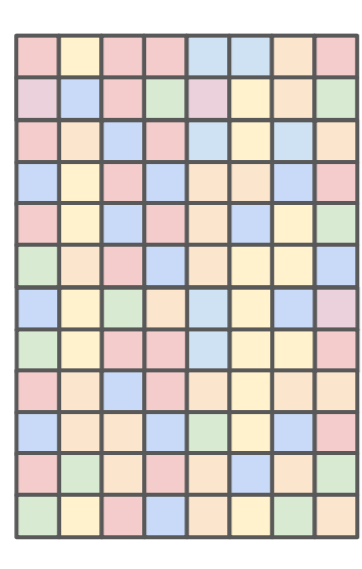
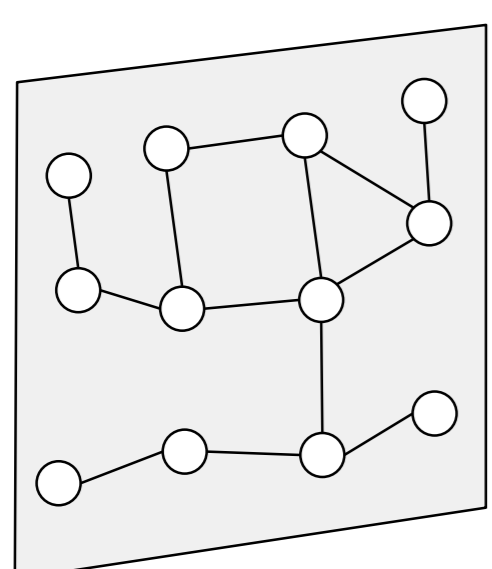
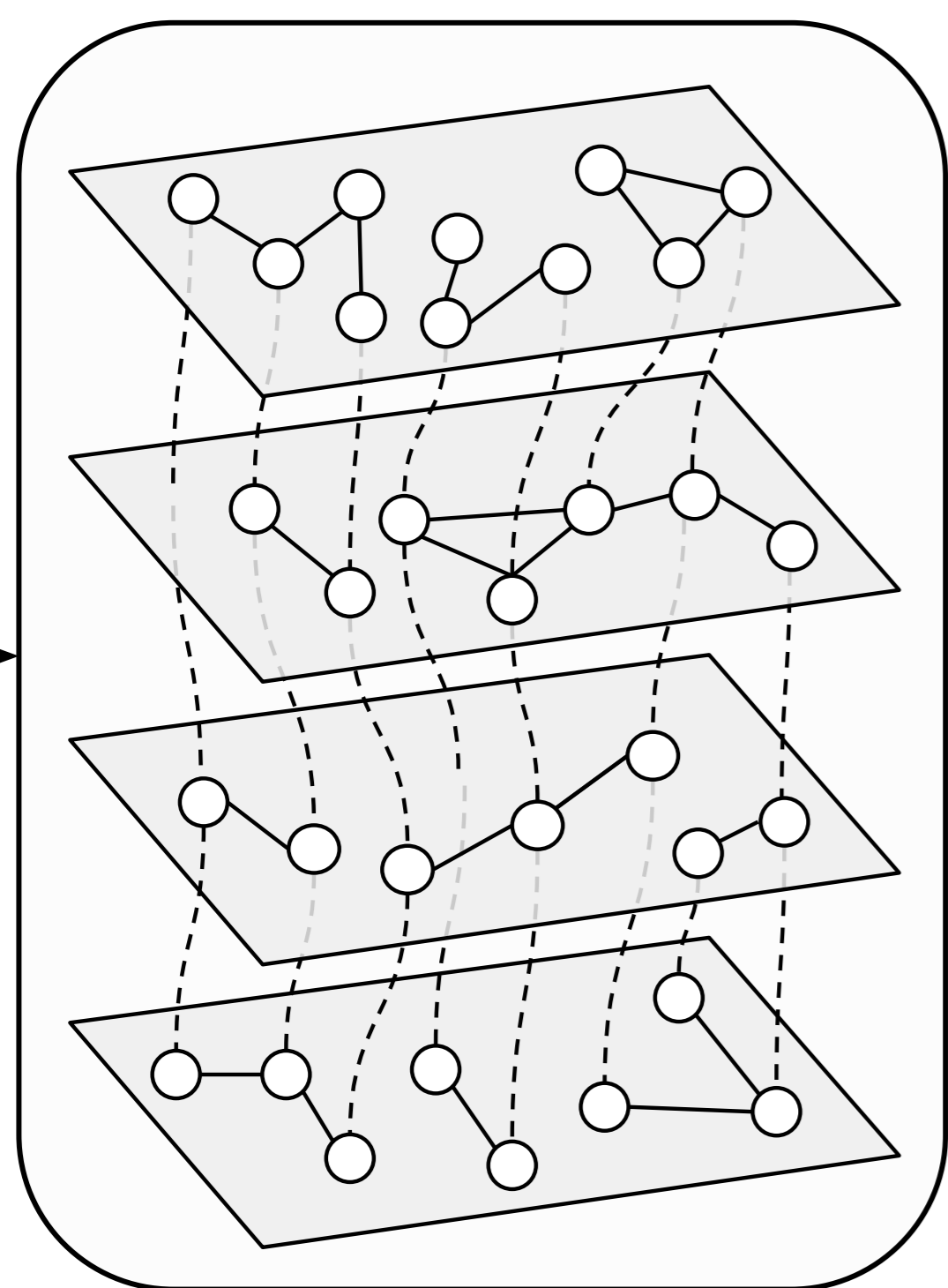
Majority of models consider **static communities**; the advantages of GNNs over principled statistical methods is not clear-cut [3]; scarcity of quality real-world data [4,5]; reliance on proxy objectives (e.g., reconstruction error) and memory-based mechanisms (GRUs, LSTM, etc.) increase computational costs and reduce interpretability.

LMoN 🍊

We propose a **Longitudinal Modularity optimization Network** based on **DMoN** [6] for **dynamic** community detection on evolving networks:

INPUT

(continuous-time temporal graph)



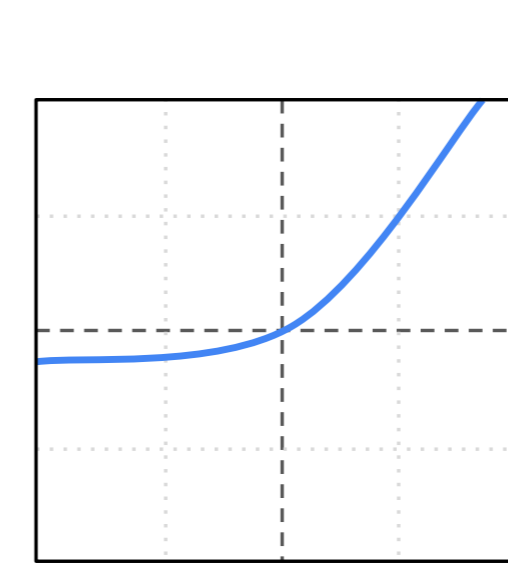
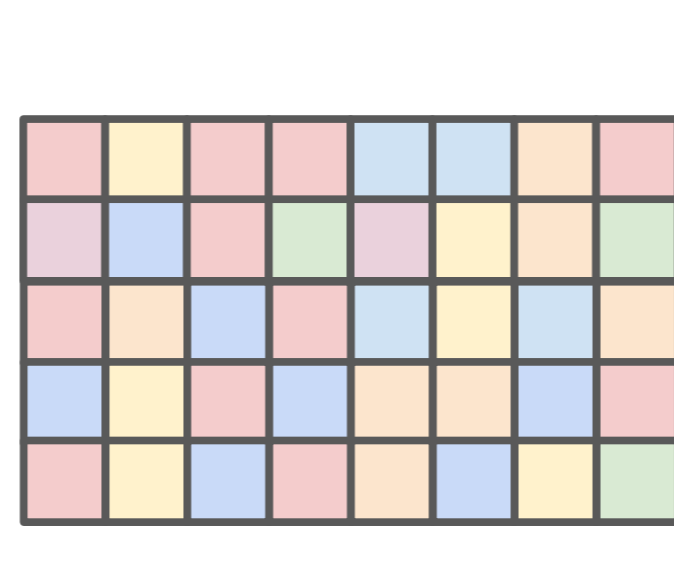
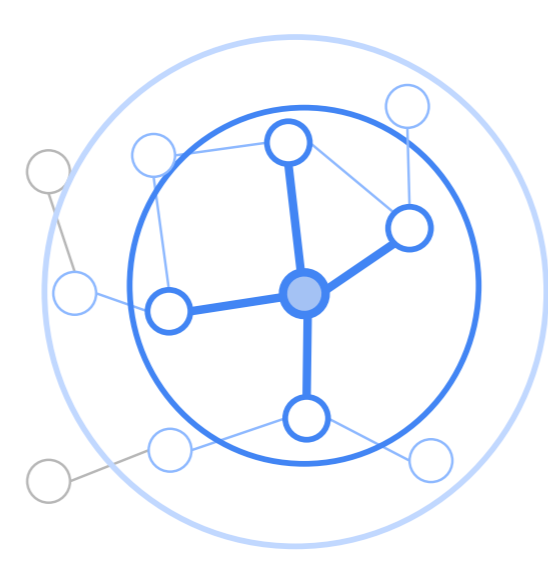
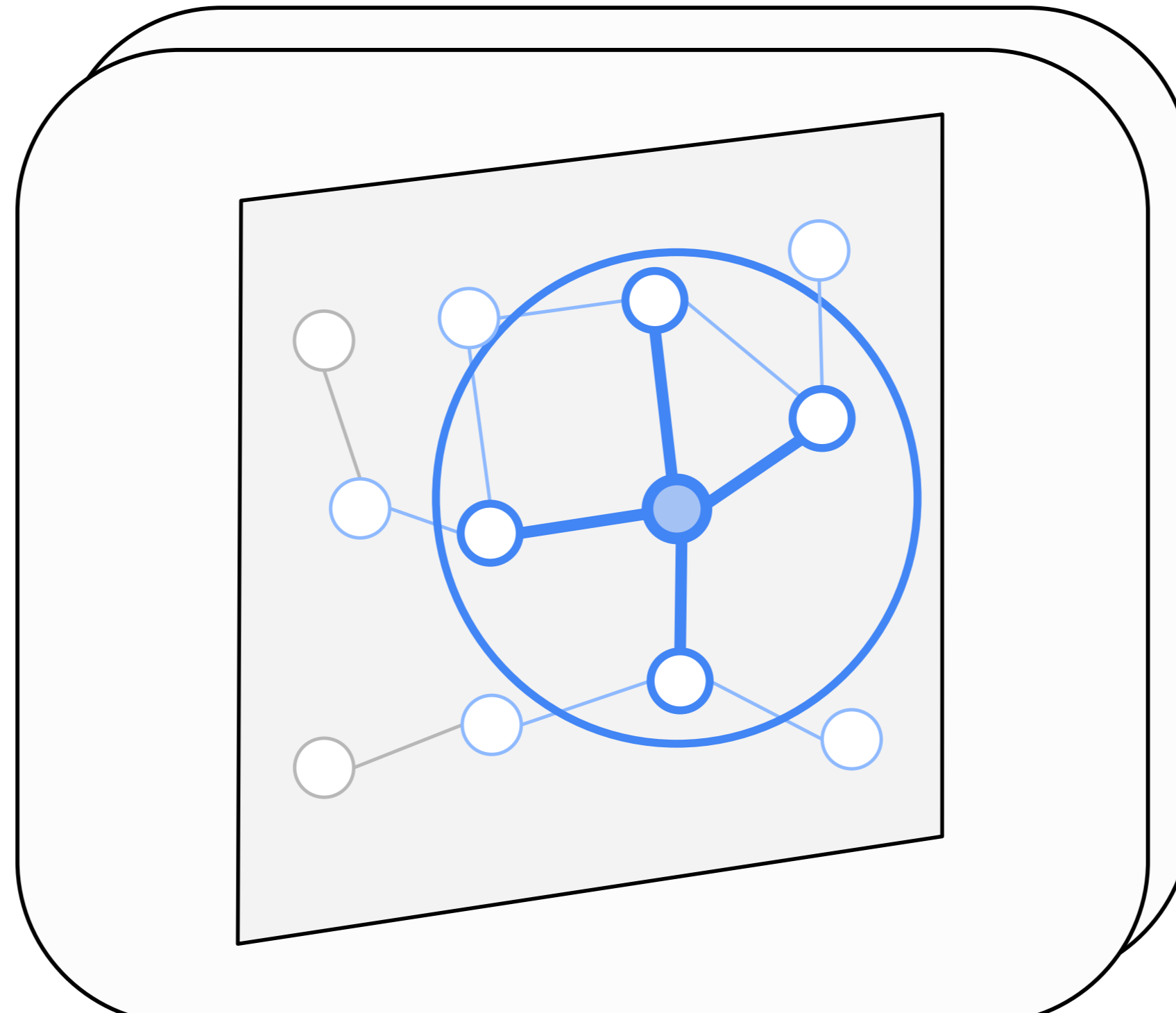
$f_h(\tilde{A})$

X

A Hawkes-Laplacian operator f_h models pairwise interaction strength in an **encoder-agnostic** way.

HIDDEN LAYER(S)

(any optionally multilayer graph encoder)



$N(u)$

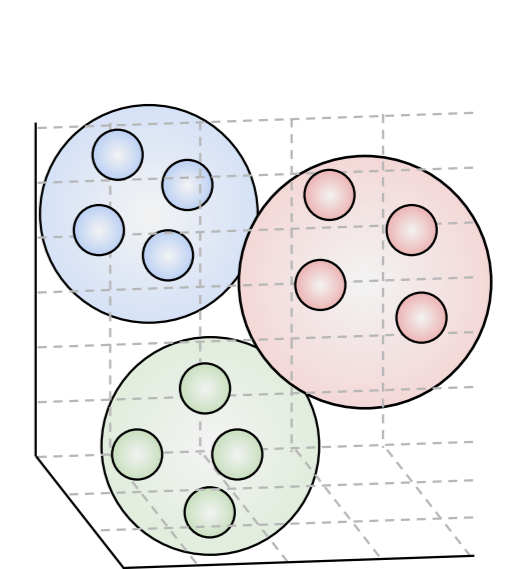
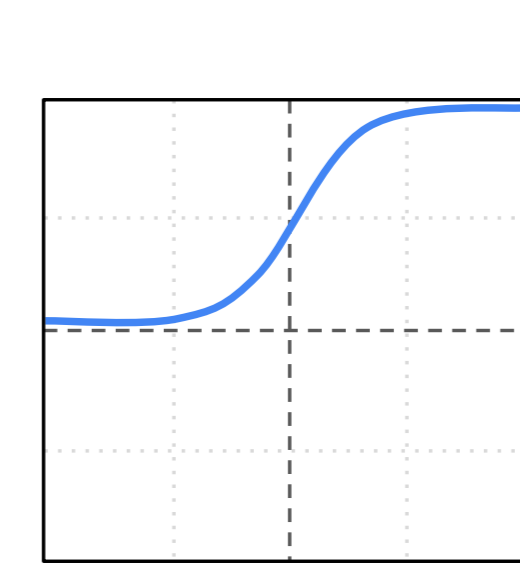
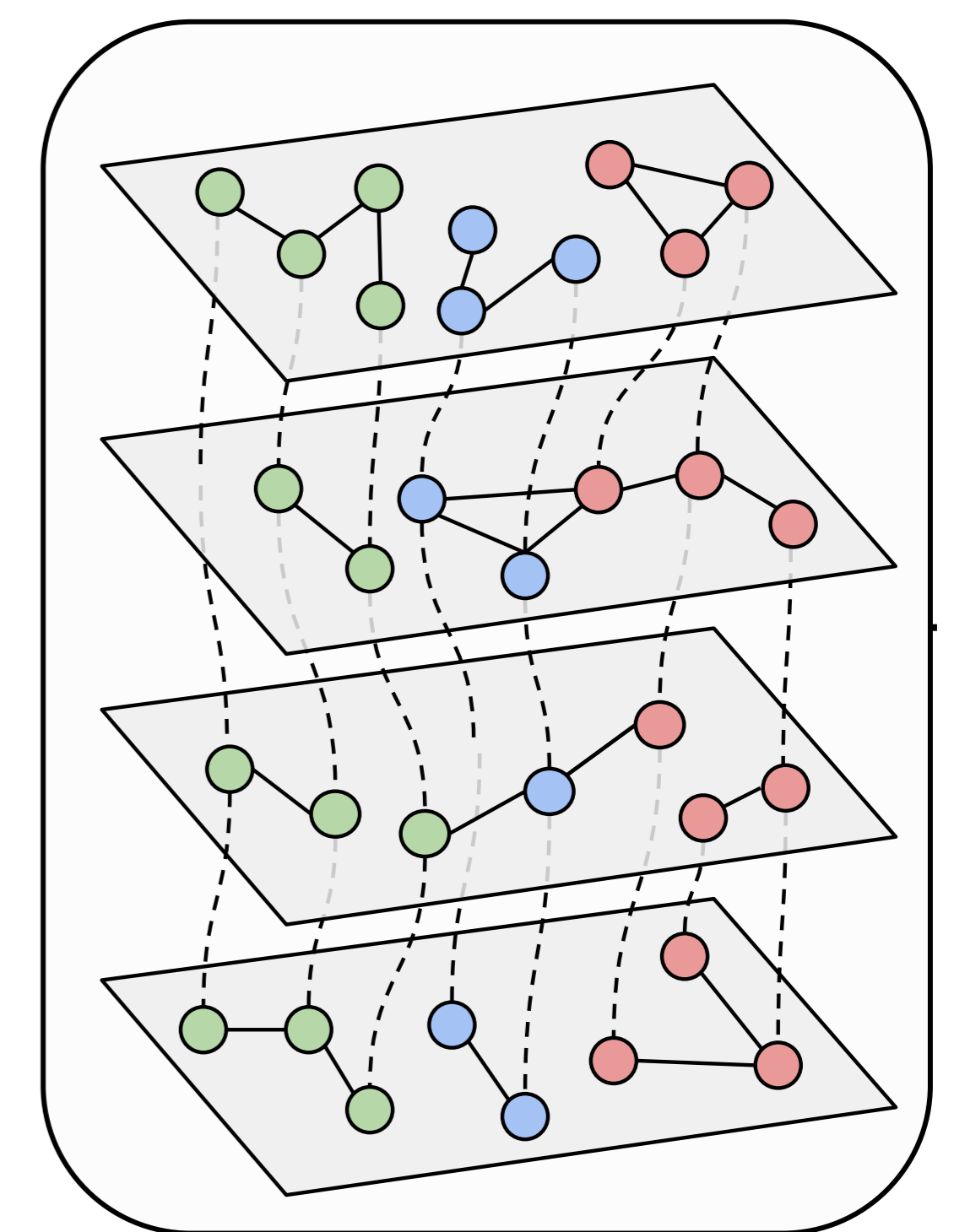
$X[N(u)]$

SeLU

A novel spectral relaxation of **longitudinal modularity** [7] for CTTGs, where a Riemann sum in the loss incentivizes contiguous label assignments.

OUTPUT

(community assignments)



Softmax

C

An unpooling layer obtains clusters from event-level representations maintaining* **best-in-class** scalability $\mathcal{O}(d^2n + m)$.

$$\mathcal{L}_{\text{LMoN}} = \underbrace{-Q + S + R}_{\text{Modularity Sm.+Reg.}} + \underbrace{H}_{\text{Hawkes NLL}}$$

(*) Our current and main **limitation**: how can we extend our graph learning pipeline for dynamic feature aggregation preserving its scalability?

[1] Temporal Network Theory. Holme & Saramäki (Org.), Springer, 2012.

[2] Community detection in networks: A user guide. Fortunato & Hric, Physics Reports, 2016.

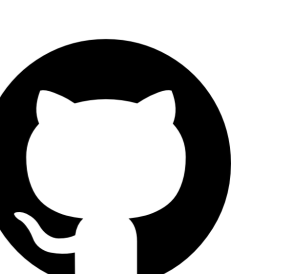
[3] Statistical inference links data and theory in network science. Peel et al., Nature Communications, 2022.

[4] The ground truth about metadata and community detection in networks. Peel et al., Science Advances, 2017.

[5] TADC-SBM: A time-varying, attributed, degree-corrected stochastic block model. Passos et al., ISCC, 2025.

[6] Graph Clustering with Graph Neural Networks. Tsitsulin et al., JMLR, 2023.

[7] Longitudinal modularity, a modularity for link streams. Brabant et al., EPJ Data Science, 2025.



Preprint coming soon! :-)